JOURNAL OF PROPULSION AND POWER
Vol. 21, No. 3, May–June 2005

Parametric Cycle Analysis of a Turbofan Engine with an Interstage Turbine Burner

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This study focuses on the parametric cycle analysis of a dual-spool, separate-flow turbofan engine with an interstage turbine burner (ITB). The ITB considered in this paper is a relatively new concept in modern jet engine propulsion. It serves as a secondary combustor and is located between the high-pressure and the low-pressure turbines, that is, the transition duct. The objective of this study is to use engine design parameters, such as high-pressure and low-pressure turbine inlet temperatures to obtain engine performance parameters, for example, specific thrust and thrust specific fuel consumption. A turbine cooling model is also included. Results confirm the advantages of ITB, that is, higher specific thrust, less cooling air, and possibly less NO_x production, provided that the main-burner exit temperature and ITB exit temperature are properly specified.

Nomenclature

a = sound speed
c_P = specific heat at constant pressure
e = polytropic efficiency
F = uninstalled thrust
f = fuel/air ratio, or function
g_c = Newton's constant
h_{PR} = low heating value of fuel
M = Mach number
$\dot{m}$ = mass flow rate
P = static pressure
P_t = total pressure
R = universal gas constant
T_t = total temperature
V = absolute velocity
α = bypass ratio
β = bleed air fraction
γ = specific heat ratio, c_p/c_v
ε = cooling air fraction
η_m = mechanical efficiency
η_{th} = thermal efficiency
π = ratio of total pressure
π_r = ratio of total pressure to static pressure due to the ram effect, P_t/P_0
τ = ratio of total temperature
τ_r = ratio of total temperature to static temperature as a result of the ram effect, T_t/T_0
τ_λ = ratio of burner exit total enthalpy to enthalpy at ambient condition

Subscripts

b = main burner
c = engine core, compressor, or properties at upstream of main burner
ci = critical ITB
f = fan
hpc = high-pressure compressor
hpt = high-pressure turbine
itb = ITB, or properties at downstream of ITB
lpc = low-pressure compressor
m = coolant mixer, or mechanical
t = properties between main burner exit and downstream, or total/stagnation values of properties
0 = engine inlet

Introduction

IN most common airbreathing propulsion engines, air is used as a medium to convert the energy available in the fuel-air mixture into propulsive thrust. Increases in pressure and momentum across the engine produce sufficient thrust to power the aircraft. Throughout aerovehicle evolution, scientists and engineers have attempted to improve engine efficiency, to make it smaller, lighter, require less fuel consumption, and yet more powerful. One of the proposed solutions to achieve these goals is the introduction of interstage turbine burner (ITB) into the engines.

ITB was also known as a reheat cycle,[1] where the expanded gas from each expansion process in a turbine is reheated before the next expansion process. Some works on reheat cycle that have recently been done are given in the literature; for example, see Vogeler,[2] Liu and Sirignano,[3] Sirignano and Liu,[4] Chen et al.,[5] and Siow and Yang.[6]

The work presented here is a parametric cycle analysis of a dual-spool, separate-exhaust turbofan engine with an ITB, which is also known as the on-design analysis. Most commercial turbofan engines have a transition duct between the high-pressure turbine (HPT) and the low-pressure turbine (LPT). The ITB considered in this study is to use transition duct as the secondary combustor. By doing so, no new engine component is added to the existing system, and fuel is burned in a moderately high-pressure environment.

The major advantages associated with the use of ITB are an increase in thrust and potential reduction in NO_x emission, as illustrated in Fig. 1. In Fig. 1a, the inlet temperature of the HPT remains unchanged. As the fluid undergoes secondary combustion, a higher specific thrust (ST) is produced. Figure 1b shows the case in which the peak temperature inside the main combustor is decreased, therefore, potentially reducing the amount of thermal NO_x production. Furthermore, by lowering the temperature of the main combustor, less amount of cooling air is required for cooling HPT blades.

The ITB engine being studied here is similar to the 1-ITB turbofan engine in Sirignano and Liu.[4] Their preliminary study showed the advantages of turbine burners (TB) for both uncooled turbojet and turbofan engines. However, without considering turbine cooling in their investigation advantages of using TB can be overestimated,

Presented as Paper 2003-0685 at the 41st Aerospace Sciences Meeting and Exhibit, Reno, NV, 6–10 January 2003; received 19 May 2003; revision received 12 November 2004; accepted for publication 13 November 2004.

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especially at a high TB inlet temperature (e.g., 1900 K). Therefore, a turbine-cooling model is included in this work to account for the thermal losses and its effect on the engine performances.[5]

Approach

Aircraft Engine Performance Parameters

In aircraft propulsion design, one of the performance parameters is the ST, which is defined as thrust F produced per unit air mass flow rate and expressed as

$$\mathrm{ST} = F/\dot{m}_{\mathrm{air}} \tag{1}$$

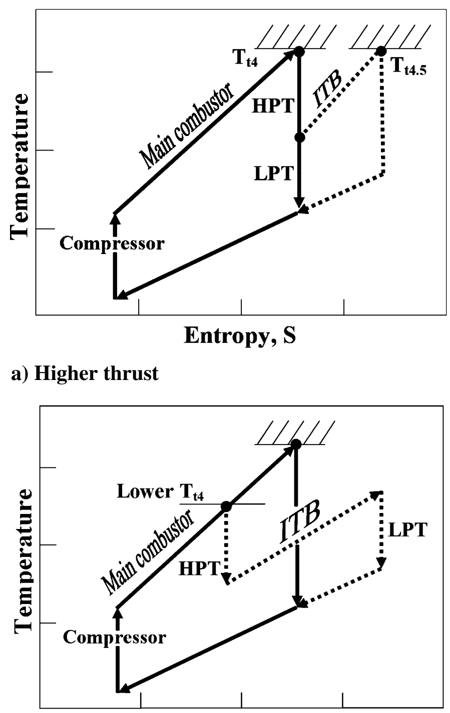
Thrust is the force produced as a result of the momentum and pressure increases across the engine. In this paper, $\dot{m}_{\mathrm{air}}$ is defined as the total inlet air mass flow rate of the engine.

The second performance parameter, thrust specific fuel consumption (TSFC), defines the rate of total mass flow rate of fuel per unit thrust produced. Accordingly,

$$\mathrm{TSFC} = \dot{m}_f/F \tag{2}$$

Another useful engine performance parameter is thermal efficiency, which is defined as the net rate of the kinetic energy gain out of the engine divided by the rate of thermal energy available from the fuel:

$$\eta_{\mathrm{th}} = \dot{E}_{\mathrm{kinetic,gain}}/(\dot{m}_f \cdot h_{\mathrm{PR}}) \tag{3}$$



a) Higher thrust

b) Reduced NO_x

Fig. 1 Thermodynamic cycles of a turbofan engine with ITB.

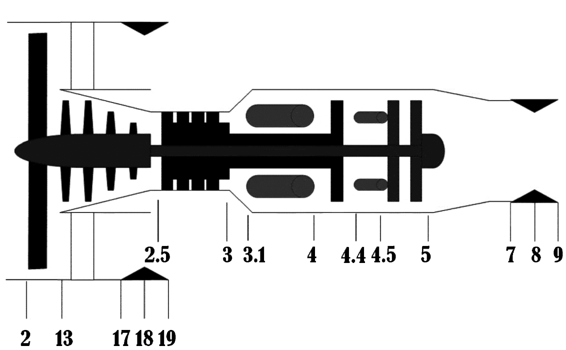


Fig. 2 Station numbering of a turbofan engine with ITB.

Assumptions

The station numbering for the turbofan cycle analysis with ITB is in accordance with Aerospace Recommended Practice (ARP) 755A (Ref. 7) and is given in Fig. 2. The ITB (i.e., the transition duct) is located between station 4.4 and 4.5.

The following assumptions are used:

1) The working fluid is air that behaves as a perfect gas with constant properties at three different sections: γ_c, R_c, C_{pc} for gas upstream of main burner (i.e., station 3.1); γ_t, R_t, C_{pt} for gas between station 4 and 4.4; γ_{itb}, R_{itb}, $C_{p\mathrm{itb}}$ for gas downstream of ITB (i.e., station 4.5).

2) All components are adiabatic.

3) Constant polytropic efficiencies[8] e of compressor, turbine, and fan will be used to relate the stage total pressure ratio π to total temperature ratio τ.

Turbine-Cooling Model

Figure 3 illustrates an engine layout with the cooling airflow paths. The approximate turbine-cooling model incorporated in this analysis was first presented in Oates.[9] A portion of the cooling air drawn off at the compressor exit is used to cool the HPT stator ε_1 and the remainder ε_2 to cool the HPT rotor. For this single-stage HPT, it is assumed that the mixing of two cooling air with mainstream occurs after the stator and rotor. Cooling air is not included for the LPT.

Figure 4 shows the correlation curves for the amount of cooling air required for each stator and rotor, as given in Walsh and Fletcher.[10] All curves are based upon the stator outlet temperature $T_{t4.2}$. For an aeroengine, the state-of-the-art maximum allowable turbine blade temperature[11] (by year 2000) is 1370 K. Therefore, it is reasonable to use the high-tech cooling curve and assume that the cooling is only required when the engine core temperature exceeds 1300 K.

Parametric Cycle Analysis

In this study, the airstream entering the turbofan engine will flow through the fan and the engine core separately. Cycle analysis is

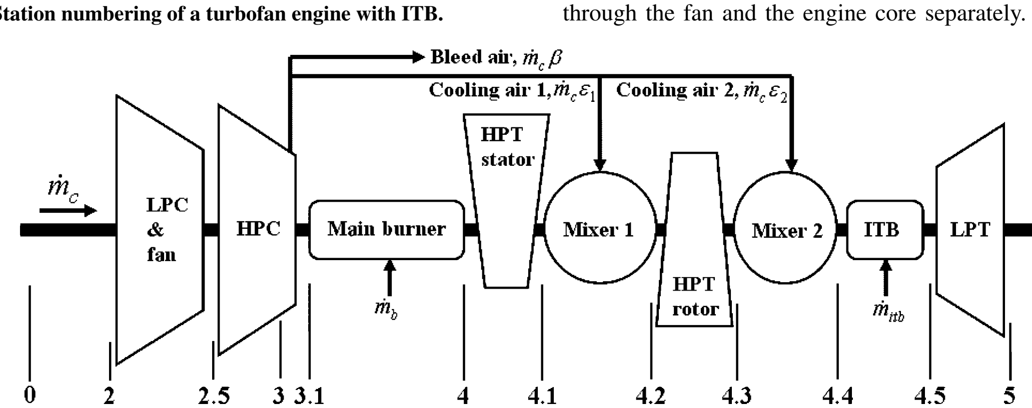


Fig. 3 Engine layout with cooling airflow.

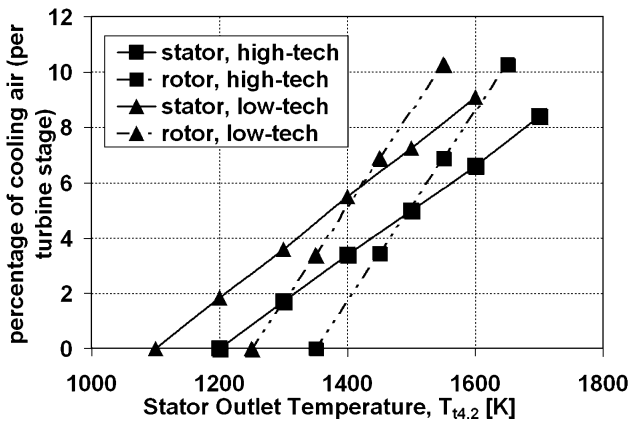


Fig. 4 Correlation curves of cooling air percentage (stator and rotor) vs stator outlet temperature $T_{t4.2}$ (Ref. 10).

then applied to both the bypass stream and engine core stream separately as listed next. The analysis follows closely as described in Mattingly.[8,12]

Bypass Stream

In this study, we consider only the uninstalled thrust F, which depends on the engine alone and hence is independent of the nacelle. Expressed in terms of the flight Mach number M_0, sound speed a_0, temperature T, and pressure P, the uninstalled thrust of bypass stream per unit fan air mass flow rate is given by

$$\frac{F_f}{\dot{m}_f} = \frac{a_0}{g_c}\left(\frac{V_{19}}{a_0} - M_0 + \frac{T_{19}/T_0}{V_{19}/a_0}\frac{1 - P_0/P_{19}}{\gamma_c}\right) \tag{4}$$

Engine Core Stream

The engine core-stream cycle analysis is similar to the bypass stream except it includes energy addition/subtraction across all components in the engine. The uninstalled thrust of the engine core stream per unit core airflow rate is given by

$$\frac{F_c}{\dot{m}_c} = \frac{a_0}{g_c}\left\{\begin{array}{l} [(1-\beta-\varepsilon_1-\varepsilon_2)(1+f_b)+\varepsilon_1+\varepsilon_2]\dfrac{V_9}{a_0} - M_0 \\ \quad + [(1-\beta-\varepsilon_1-\varepsilon_2)(1+f_{\text{itb}})+\varepsilon_1+\varepsilon_2](1+f_{\text{itb}}) \\ \quad \times \dfrac{R_{\text{itb}}}{R_c}\dfrac{T_9/T_0}{V_9/a_0}\dfrac{1}{\gamma_c}\left(1-\dfrac{P_0}{P_9}\right) \end{array}\right\} \tag{5}$$

where f_b and f_{itb} are the fuel-air ratios in the main combustor (MB) and in the ITB, respectively. β, bleed air fraction required for aircraft cabin pressurization, is set to zero.

Application of the steady flow energy equation to the MB yields

$$f_b = \frac{\tau_{\lambda-b} - \tau_r\tau_d\tau_c}{h_{\text{PR}-\text{b}}\eta_b/c_{\text{pc}}T_0 - \tau_{\lambda-b}} \tag{6}$$

where $\tau_{\lambda-b}$ is the ratio of the MB exit total enthalpy to the ambient enthalpy:

$$\tau_{\lambda-b} = \frac{c_{\text{pt}}T_{t4}}{c_{\text{pc}}T_0} \tag{7}$$

Similarly, application of the steady flow energy equation to the ITB yields

$$f_{\text{itb}} = \frac{\tau_{\lambda-\text{itb}} - \tau_{\lambda-b}\tau_{m1}\tau_{m2}\tau_{\text{hpt}}}{h_{\text{PR}-\text{itb}}\eta_{\text{itb}}/c_{\text{pc}}T_0 - \tau_{\lambda-\text{itb}}} \tag{8}$$

where $\tau_{\lambda-\text{itb}}$ is the ratio of the ITB exit total enthalpy to the ambient enthalpy. τ_{m1} and τ_{m2} are the total temperature ratio across the coolant mixer 1 and 2, respectively.

$$\tau_{\lambda-\text{itb}} = \frac{c_{p\text{itb}}T_{t4.5}}{c_{\text{pc}}T_0} \tag{9}$$

For a dual-spool turbofan engine, high-pressure compressor (HPC) and HPT are connected through a single shaft. The total temperature ratio across the HPT is expressed as

$$\tau_{\text{hpt}} = 1 + \frac{1 - \tau_{\text{hpc}}}{(1+f_b)(C_{\text{pt}}/C_{\text{pc}})\tau_{\text{hpc}}\tau_b\eta_{\text{m}-\text{hpt}}} \tag{10}$$

where τ_{hpc} is the total temperature ratio across HPC.

On the other hand, the LPT, low-pressure compressor (LPC), and fan are connected through another shaft. Thus, the powers are balanced individually (two-unmixed-spool analysis). The power amounts between the HPC and the LPC are not specified directly. The compressor and LPC total pressure ratios are two of the independent engine design variables, whereas HPC pressure ratio is obtained from the following relation:

$$\pi_{\text{hpc}} = \pi_c/\pi_{\text{lpc}} \tag{11}$$

Turbofan Engine Performance

ST is defined as the total uninstalled thrust (through core engine and fan) per unit air mass flow rate,

$$\text{ST} = \frac{F_c/\dot{m}_c + \alpha(F_{\text{fan}}/\dot{m}_{\text{fan}})}{1+\alpha} \tag{12}$$

where α is the fan bypass ratio (FBR):

$$\alpha = \dot{m}_{\text{fan}}/\dot{m}_c \tag{13}$$

TSFC is defined as the total fuel flow rate (MB and ITB) per unit thrust produced,

$$\text{TSFC} = \frac{f_b + f_{\text{itb}}}{\text{ST}} \tag{14}$$

Computer Code and Engine Configurations

A program was written in combination among Microsoft® Excel spreadsheet neuron cells, Visual Basic, and macrocode to provide a user-friendly interface so that the compilation and preprocessing are not needed.

Engine configurations used include a base engine without ITB and ITB engines with different values of LPT inlet temperature ($T_{t4.5}$ or ITB exit temperature). The maximum value of $T_{t4.5}$ is 1300 K so that no cooling air is required for the LPT. The input data for these engines are listed in Table 1. The computed result for the base engine using Excel program was compared to the ONX program developed by Mattingly,[12] and the comparison was found to be consistent.

Results and Discussions

Effect of Turbine Cooling

Figure 5 shows the performance comparison for the base turbofan engine with and without turbine cooling at flight Mach number of 0.0 to 2.0. ST of the uncooled base engine is overestimated by around 10% at subsonic flight ($M = 0.9$) and 15% at supersonic flight ($M = 1.5$). Therefore, a turbine-cooling model is required to account for the effect of turbine cooling on the engine performance.

Effect of HPT and LPT Inlet Temperature

Figures 6a–6c compare the engine performances of the base engine with four different ITB engines for HPT inlet temperatures T_{t4} from 1000 to 2000 K. In Sirignano and Liu,[4] it is clearly shown that the TB engines benefits from higher T_{t4} at both subsonic and supersonic flights. Similarly, as the value of T_{t4} increases, all engines (at Mach number of 0.85) exhibit an increase in both ST and thermal efficiency, accompanied by a decrease in TSFC as shown in Fig. 6.

Data show that whenever the gain in ST is greater than the loss caused by extra fuel consumption in ITB it leads to a better thermal

Table 1 Engine input parameters

Description	Input value
Design parameters (for base engine)	
CPR π_c	28.48
Flight Mach number M	0.85
FBR α	4.0
FPR π_f	1.3
LPC pressure ratio π_{lpc}	1.387
MB exit temperature T_{t4}	1600 K
Polytropic efficiencies	
Fan e_{fan}	0.93
HPC e_{hpc}	0.9085
HPT e_{hpt}	0.8999
LPC e_{lpc}	0.8738
LPT e_{lpt}	0.9204
Total pressure ratios	
Inlet $\pi_{\text{d,max}}$	0.99
MB π_b	0.96
ITB π_{ITB}	0.96
Nozzle π_n	0.99
Fan nozzle π_{fn}	0.98
Component efficiencies	
MB η_b	0.99
ITB η_{ITB}	0.99
Fuel low heating value h_{PR}	43124 kJ/kg
Bleed air fraction β	0

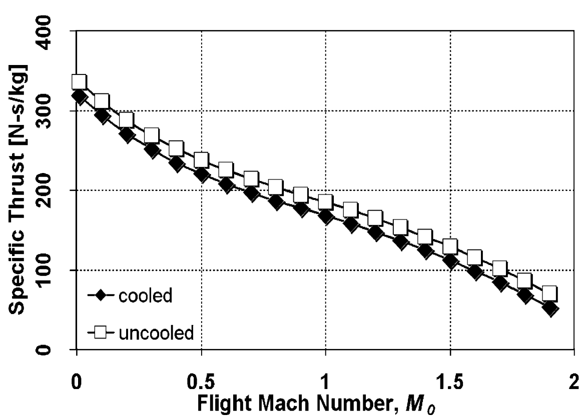


Fig. 5 Performance comparison of base engines vs M_0, $\pi_f = 1.3$, $\pi_c = 28.48$, $T_{t4} = 1600$ K, and $\alpha = 4.0$.

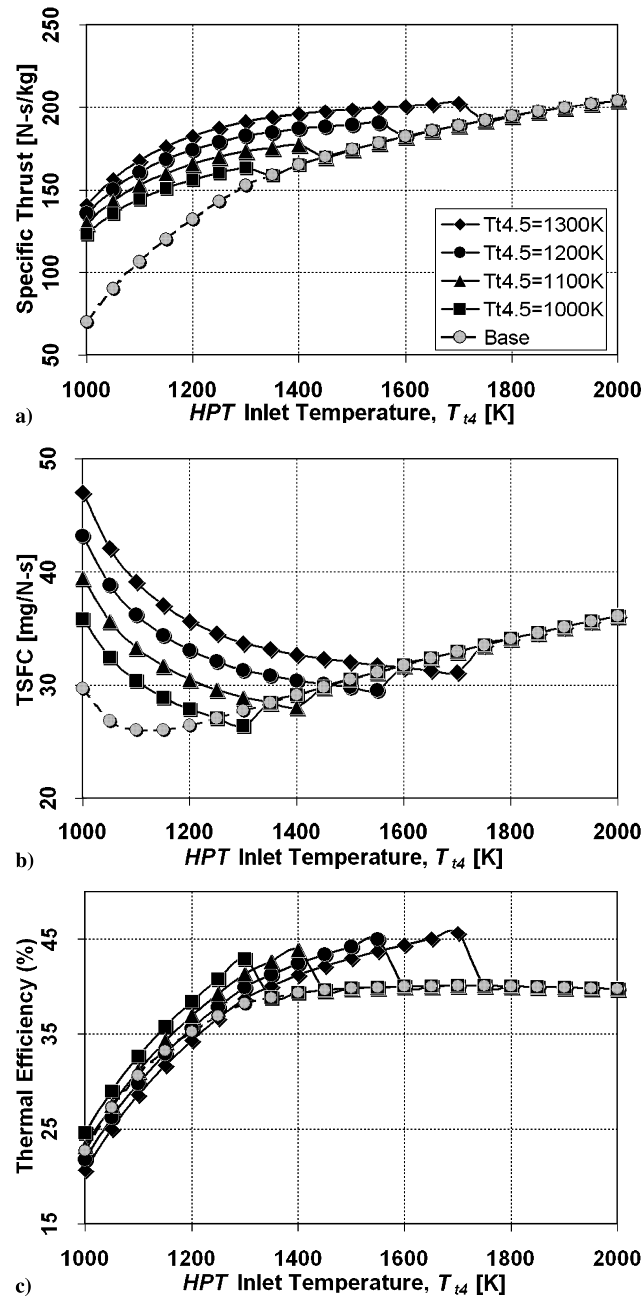


Fig. 6 Performances of cooled turbofan engines vs T_{t4} at $M_0 = 0.85$, $\pi_f = 1.3$, $\pi_c = 28.48$, and $\alpha = 4.0$.

efficiency. Clearly, ITB engines always generate higher ST and gain better thermal efficiency than the base engine. Nevertheless, this is not the case when T_{t4} is lower than $T_{t4.5}$, in which a large portion of fuel is now burned in the ITB (a relatively lower pressure environment compared to main burner); therefore, thermal efficiency will drop below that of the base engine. The thermal efficiency curves in the region of lower HPT inlet temperature (i.e., T_{t4} lower than 1300 K) clearly reflect this phenomenon.

However, for all ITB engines, each ST and thermal efficiency curve will drop and merge with the base engine curve after reaching a maximum point at some values of T_{t4}. These maximum points in the ST curves and thermal efficiency curves correspond to the minimum points in TSFC curves. We refer to these values of T_{t4} as critical ITB temperatures T_{ci}, beyond which the ITB will be turned off, resulting in a discontinuity on each performance curve. One can explain this by looking at Eq. (8). It is possible to yield a negative value of f_{itb} when the total enthalpy ratio after the expansion across HPT is greater than the ITB exit total enthalpy ratio [see Eq. (15) as shown next].

$$\tau_{\lambda-\text{itb}} - \tau_{\lambda-b}\tau_{m1}\tau_{m2}\tau_{\text{hpt}} < 0 \tag{15}$$

At such a condition, no further energy addition is allowed in ITB. Even worse, a negative value of f_{itb} means that energy needs to be extracted from ITB, which is considered a counteraction of adding ITB, and it is not desirable. Unfortunately, this also tells us that the ITB engines at T_{t4} higher than T_{ci} will not benefit from the advantages of ITB, unless $T_{t4.5}$ is further increased. For instance, a base engine with T_{t4} of 1600 K will only benefit from adding ITB if $T_{t4.5}$ is set to 1300 K (see Fig. 6) because its corresponding value of T_{ci} (i.e., 1713 K as shown in Fig. 7) is greater than T_{t4} of 1600 K. Therefore, it is always desirable to seek a higher T_{ci} in the engine design process in order to utilize the advantage of ITB. An example is in Sirignano and Liu's study,[4] where T_{06} (equivalent to $T_{t4.5}$ in this paper) is specified at 1900 K, much higher than T_{04} (equivalent to T_{t4}) of 1500 K. This explains why T_{ci} is never seen in their analysis. As mentioned earlier, one concern of turbine cooling is the reduction in engine performance. Having a high value of $T_{t4.5}$ (i.e., 1900 K), turbine cooling is also required for the LPT, which might offset the advantage of using ITB.

To predict T_{ci}, Eq. (15) is rewritten as shown in Eq. (16):

$$\tau_{\lambda-b}\tau_{m1}\tau_{m2}\tau_{\text{hpt}} = \tau_{\lambda-\text{itb}} \tag{16}$$

where τ_{m1}, τ_{m2}, and, τ_{hpt} are mainly functions of $\tau_{\lambda-b}$ [see Eq. (7)].

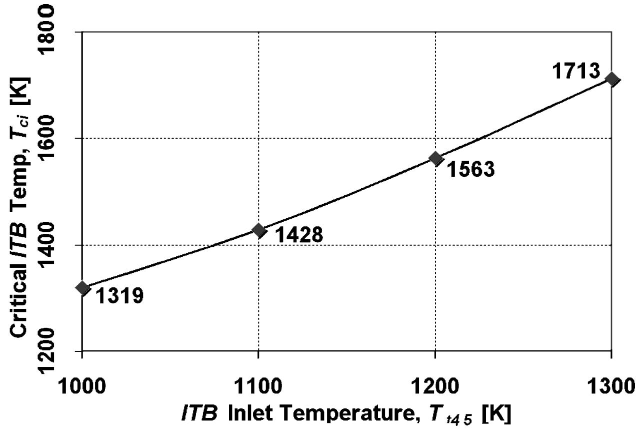


Fig. 7 Variation of critical ITB temperature T_{ci} with $T_{t4.5}$ at $M_0 = 0.85$, $\pi_f = 1.3$, $\pi_c = 28.48$, and $\alpha = 4.0$.

The value of T_{t4} that satisfies Eq. (16) is referred as T_{ci}. Equation (16) is solved by an iterative procedure, and the result is shown in Fig. 7. It is found that the higher the value of $T_{t4.5}$, the higher the value of T_{ci}. This is because a higher $T_{t4.5}$ allows more heat addition in ITB.

Effect of Flight Mach Number

Figures 8a–8c demonstrate the performance comparisons for the base turbofan engine and four different ITB engines at flight Mach number of 0.0 to 2.0. For each engine, the compressor pressure ratio (CPR), fan pressure ratio (FPR), and FBR are fixed at 28.48, 1.3, and 4.0, respectively, with a maximum allowable T_{t4} of 1600 K. All types of engines exhibit a decrease in ST and an increase in TSFC as flight Mach number increases. These trends of performance are similar to the findings of Sirignano and Liu.[4]

Clearly, the ITB engine with $T_{t4.5}$ of 1300 K (the highest $T_{t4.5}$ among all other ITB engines) performs better than the base engine at both subsonic and supersonic flights. There is a rise in both ST and thermal efficiency without any increase in TSFC. ITB engines

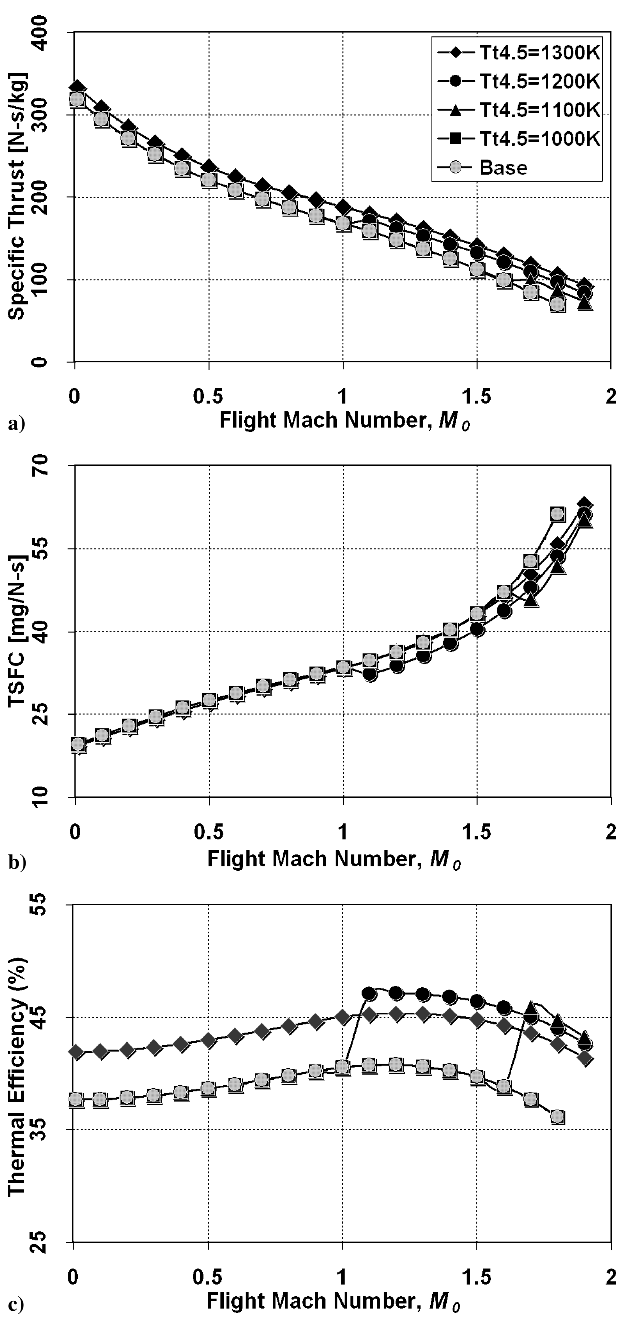


Fig. 8 Performances of cooled turbofan engines vs M_0, $\pi_f = 1.3$, $\pi_c = 28.48$, $\alpha = 4.0$, and $T_{t4} = 1600$ K.

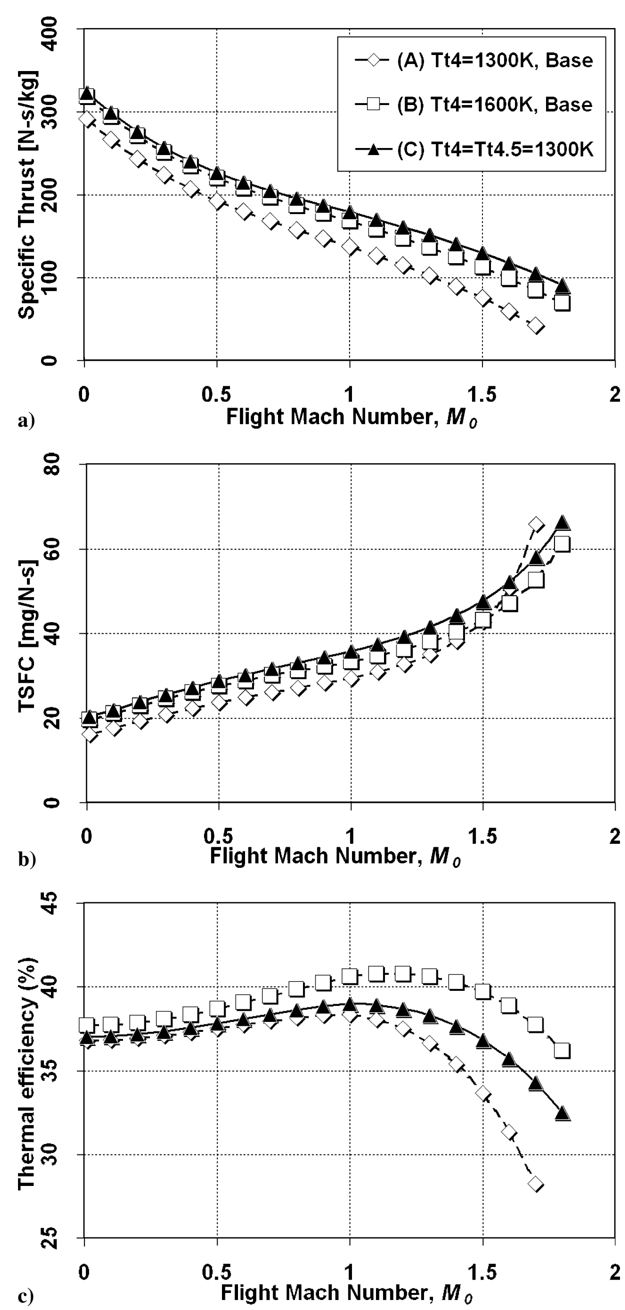


Fig. 9 Performances of three cooled turbofan engines vs M_0, $\pi_f = 1.3$, $\pi_c = 28.48$, and $\alpha = 4.0$.

with lower values of $T_{t4.5}$ (i.e., 1200 and 1100 K) are beneficial at supersonic flight only. On the contrary, an ITB engine with the lowest value of $T_{t4.5}$ (i.e., 1000 K) gains no improvement at all. One reason is that this ITB engine is operating at T_{t4} (i.e., 1600 K) greater than the T_{ci} (i.e., 1319 K at M_0 of 0.85). [The effect of flight Mach number on T_{ci} is considered negligible because T_{ci} only increases slightly with increasing flight Mach number (data not shown here).] One can argue to use a higher value for $T_{t4.5}$. However, at higher $T_{t4.5}$, especially when it is greater than the blade material limit, turbine cooling is required for LPT. This is unfavorable because more air would be extracted from LPC to cool down LPT, which might offset any advantage of having higher $T_{t4.5}$. Therefore, in order to avoid cooling LPT, the maximum $T_{t4.5}$ will be the turbine blade material limit (i.e., 1300 K in this study).

Three engine design choices, namely CPR, FPR, and FBR, are also studied. Despite using a different approach and engine configuration, this study yields similar conclusions as Sirignano and Liu's findings.[4] Therefore, the results are not shown here.

Combined Advantages

An important target in the engine design with ITB is the reduced amount of thermal NO_x. One way to achieve this is to lower the peak temperature inside the burner. We first consider three engines in the following discussion: engine A (i.e., a base engine with T_{t4} set to 1300 K), engine B (i.e., a base engine with T_{t4} set to 1600 K), and engine C (i.e., engine A with an addition of ITB, where $T_{t4.5}$ is set to 1300 K). Figures 9a–9c demonstrate the performance comparisons of these three engines for flight Mach number of 0 to 2.

In the preceding discussions, it becomes clear that as the fluid undergoes secondary combustion a higher ST results with an improvement in thermal efficiency. Therefore, it is expected to see engine C having better ST and improved thermal efficiency than engine A. On the other hand, despite having a lower efficiency as a result of a lower peak temperature (i.e., 1300 K) inside the burners, engine C has advantages over engine B. Its advantages include higher ST, lesser amount of NO_x production, and no turbine cooling required. Small amounts of cooling air might be required if a low-technology blade material is used, or if higher values of T_{t4} and $T_{t4.5}$ are specified. Although engine C is said to produce a lesser amount of thermal NO_x as a result of the lower peak temperatures inside the burners, further study is required to verify this.

Conclusions

Results of the parametric studies presented in this paper can be summarized as follows:

1) Turbine cooling is essential for predicting the engine performance.[5]

2) It is important to identify the critical ITB temperature T_{ci} for each engine configuration, beyond which the ITB has no advantage at all. Therefore, it is always desirable to seek for a higher T_{ci} in order to utilize the advantage of ITB.

3) The results confirmed the advantage of using ITB, that is, higher ST, improved thermal efficiency, less cooling air, and possibly less NO_x production, provided that the values of T_{t4} and $T_{t4.5}$ are properly specified.

Despite many advantages of using ITB for better engine performance, there are also challenges needed to be resolved. Specific hardware design challenges are the design and integration of a second combustor, including all associated cooling and control requirements, which need to be overcome.

The current computer program is written for a specific engine configuration, namely, unmixed two-spool turbofan engine with a separate fan and engine core-stream nozzles. Application of an ITB is not limited to this configuration, and additional options for different engine configurations would be a very desirable feature. A research on the performance cycle analysis (off design) of a turbofan engine with an ITB is ongoing.

Acknowledgment

The authors would like to thank NASA Glenn Research Center for its financial support, especially Cecil J. Marek and Paul F. Penko. The authors would also like to thank Scott Jones of NASA Glenn Research Center, Jeffrey Herbon of Williams International, Feng Liu of the University of California, Irvine, and Jack D. Mattingly of Mattingly Consulting, for their valuable and instructive comments.

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